On the Plumbing Structure of Fibre Surfaces
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Abstract. This thesis is situated in the mathematical field of low-dimensional topology and is concerned with a particular class of knots and links known as fibred links. A link \( L \) in the three-sphere is fibred if its complement admits a fibration over the circle with fibres the interiors of Seifert surfaces for \( L \). These surfaces, called fibre surfaces, are our main objects of study. For example, all torus links and all positive braid links are fibred. The simplest example of a fibred link is the Hopf link; its fibre surface is an annular band with a full twist. Using a geometric operation called plumbing, Hopf bands can be glued together so as to form more complicated fibre surfaces. The reverse operation, deplumbing, consists in removing a Hopf band from a fibre surface. It amounts to cutting that fibre surface along a proper arc.

By a theorem of Giroux and Goodman’s, every fibre surface has a plumbing structure: it can be obtained from the standard disk by plumbing and deplumbing some number of Hopf bands. Our main results concern the (non-)uniqueness of the plumbing structure of a given fibre surface. Specifically, we study the cases of torus links and positive arborescent Hopf plumbings. Among these, we characterise the links arising from simple plane curve singularities by the finiteness of proper arcs that correspond to deplumbing a Hopf band.